

Do Incomplete GUT Multiplets Always Spoil Unification?

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Abstract. We consider a new class of light vectorlike exotics with fractional electric charge which do *not* come in complete representations of a grand unified gauge group, and are nevertheless compatible with gauge coupling unification and other predictions from Grand Unified Theories. Such states naturally arise in orbifold constructions of the heterotic string. Some aspects of their phenomenology and the consequences for the LHC are explored.

Submitted for the SUSY07 proceedings. Based on Phys. Rev. Lett. **99**, 051802 (2007) in collaboration with Stuart Raby.

PACS. PACS-key 11.25.Mj, 11.25.Wx, 12.10.Kt

1 Motivation

1.1 The Standard Model and Beyond

There is no convincing experimental data in disagreement with the Standard Model (SM), and yet we have good reasons to believe in new physics beyond the scale of a few TeV: The Standard Model has 26-28 parameters (including Dirac or Majorana neutrinos) which seemingly take arbitrary values. There is no explanation for the observed pattern of gauge symmetries and no organizing principle for the particles in each generation, and also no reason why these particles come in 3 generations at all. The mass of the only scalar particle in the theory, the yet-to-be-discovered Higgs boson, receives large radiative corrections and requires an incredible fine-tuning to be of order the weak scale. Over the last decade we learned from cosmology that Standard Model particles can account only for $\sim 4\%$ of the matter and energy content of the universe, leaving it open for speculation what may constitute $\sim 23\%$ which is known to be non-baryonic matter.

1.2 Grand Unification and Supersymmetry

Are there hints at physics beyond the Standard Model? When we extrapolate the couplings measured at the

Table 1. Spectrum of the (Minimal Supersymmetric) Standard Model. All states other than these are termed “exotic”.

Q	$(\mathbf{3}, \mathbf{2})_{1/3}$	L	$(\mathbf{1}, \mathbf{2})_{-1}$	H	$(\mathbf{1}, \mathbf{2})_1$
\bar{u}	$(\bar{\mathbf{3}}, \mathbf{1})_{-4/3}$	\bar{e}	$(\mathbf{1}, \mathbf{1})_2$	\bar{H}	$(\mathbf{1}, \mathbf{2})_{-1}$
\bar{d}	$(\bar{\mathbf{3}}, \mathbf{1})_{2/3}$	$\bar{\nu}$	$(\mathbf{1}, \mathbf{1})_0$		

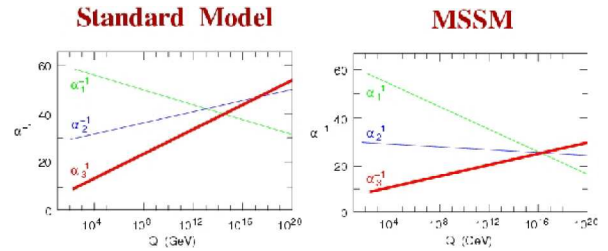


Fig. 1. The renormalization group running of the gauge couplings in the case of the SM and MSSM, respectively.

weak scale ~ 200 GeV to higher energies, they *seem to meet* at one point. It turns out that in the case of the SM, the couplings miss each other, whereas in the case of the Minimal Supersymmetric Standard Model (MSSM), they *do unify* within the experimental and theoretical uncertainties (see Fig.1). This is a strong indication not only for the existence of a Grand Unified Theory (GUT) at the scale of $\sim 3 \times 10^{16}$ GeV (M_{GUT}), but also for supersymmetry (SUSY) with superpartner masses ~ 1 TeV [1, 2].

1.3 Exotic Particles

By definition, all particles which are not in the MSSM spectrum (see Tab. 1) are termed *exotics*. Most theories beyond the Standard Model predict the existence of exotic particles. Since no exotics have been observed so far, they must necessarily be heavier than the electroweak scale¹.

The running of the gauge coupling constants sensitively depends on the spectrum of the theory. Any

¹ The exact mass bounds depend on the quantum numbers of the exotic particles, their production mechanism, and other factors. See Ref. [4] current values.

exotic particles between the electroweak and M_{GUT} will *generally* spoil this nice picture of unification, *unless* these new particles come in complete representations of the unified gauge group. The simplest example would be an $\text{SU}(5)$ as the group at the unification scale and a pair of $\mathbf{5} + \bar{\mathbf{5}}$ as exotic particles, which decompose into the SM representations

$$\begin{array}{ccccccc} \mathbf{5} + \bar{\mathbf{5}} & \rightarrow & (\mathbf{3}, \mathbf{1})_{-2/3} & + & (\mathbf{1}, \mathbf{2})_1 & + & (\mathbf{3}, \mathbf{1})_{2/3} + (\mathbf{1}, \mathbf{2})_{-1} \\ \bar{d}^c & + & L^c & + & \bar{d} & + & L \end{array} \quad (1)$$

In this case, the exotic particles are a fourth-generation \bar{d} , L , and their charge conjugates \bar{d}^c , L^c . (All fields are left-chiral. See Tab. 1 for the nomenclature.) If e.g. \bar{d} and \bar{d}^c were the only “extra” particles in the theory, the gauge couplings would not unify.

1.4 Vectorlike Pairs

Gauge symmetries do not allow for explicit mass terms for chiral particles in the Lagrangian, and the well-known Higgs mechanism generates an effective mass of order the electroweak scale. For particles to naturally acquire a mass well above the electroweak scale, they must come in *vectorlike pairs*, i.e. each state χ must be accompanied by its charge conjugate χ^c . The mass term

$$M_{\chi\chi^c} \quad (2)$$

is then gauge invariant and can be much larger than 200 GeV. In general, there are 2 arguments which seem to indicate that this mass is rather of order the GUT scale. First, M_{GUT} is the only scale in the theory, so it is “natural” to expect M to be of the same order. Second, any particles below M_{GUT} contribute to the running of the gauge couplings and may spoil unification.

The first argument is aesthetic in nature and more of a guideline for model builders than a real constraint. The second one confronts theory with data: If we believe in unification, there must not be any exotic particles lighter than M_{GUT} , which affect the GUT relations.

It is widely assumed that the only exotic particles which can be lighter than M_{GUT} and do not change the predictions from Grand Unification are those in complete representations of the GUT group.

In this study (see also Ref. [3]), we argue that there is a more general class of exotics which are *not in complete representations* of the GUT group and nevertheless *do not change most of the predictions* from Grand Unification. In fact, their only effect is to increase the value of the gauge coupling α_{GUT} at the GUT scale.

2 A New Class of Exotic Particles

2.1 The Renormalization Group Equations

The renormalization group (RG) running of the gauge couplings at one loop is given by

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_{\text{GUT}}} - \frac{1}{2\pi} b_i \log \left(\frac{M_{\text{GUT}}}{\mu} \right), \quad (3)$$

where the $i = 1, 2, 3$ refers to the gauge groups $\text{U}(1)_Y$, $\text{SU}(2)_L$ and $\text{SU}(3)_c$, respectively, and μ is the scale where the experiment actually measures the values of the gauge couplings, i.e. in most cases $\mu = M_Z$. The b_i are the β -function coefficients to be introduced below.

The β -function for a general gauge theory is given by the famous formula [5, 6]

$$\beta(g) = -\frac{1}{16\pi^2} \left[\frac{11}{3} \ell(\text{vector}) - \frac{2}{3} \ell(\text{Weyl fermion}) - \frac{1}{6} \ell(\text{spinless}) \right] g^3 + \dots, \quad (4)$$

where the dots indicate higher order corrections. In supersymmetric theories, this expression simplifies² to

$$\beta(g) = -\frac{1}{16\pi^2} \overbrace{[3\ell(\text{vector}) - \ell(\text{chiral})]}^{b_i} g^3 + \dots, \quad (5)$$

where “vector” and “chiral” denote the respective supermultiplets and $\ell(\dots)$ is the index of the representation [7]³,

$$\ell(\Lambda) = \frac{1}{2} \frac{\dim(\Lambda)}{\dim(\mathfrak{g})} (\Lambda, \Lambda + 2\delta) \quad (6)$$

given in terms of the gauge group \mathfrak{g} , the highest weight Λ of the representation in the Dynkin basis, and $\delta = (1, \dots, 1)$. The angular brackets $\langle \cdot, \cdot \rangle$ denote the scalar product.

2.2 The New Particles

For the MSSM particle content (including the gauge bosons which we have not listed in Tab. 1), Eqs. (5-6) give

$$b_1 = -\frac{33}{5}, \quad b_2 = -1, \quad b_3 = 3, \quad (7)$$

from which e.g. gauge coupling unification (see Fig. 1) follows.

Consider now the exotic particles given in Tab. 2(a). We will denote their contribution to the b_i by Δb_i , and, as an example, calculate Δb_3 . In this case, the gauge group \mathfrak{g} is $\text{SU}(3)$ and the only representations which contribute are \tilde{Q} and \tilde{Q}^c in Tab. 2(a). The highest weight of e.g. \tilde{Q} is $\Lambda = (1 \ 0)$, and keeping in mind that the Dynkin scalar product is given by the quadratic form which is the inverse of the Cartan matrix [8], Eq. (6) yields

$$\Lambda(\tilde{Q}) = \frac{1}{2} \cdot \frac{3}{8} \cdot (1 \ 0) \frac{1}{3} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \frac{1}{2}. \quad (8)$$

² There is a gaugino for each vector boson and 2 real bosons for each Weyl fermion, so combining the first and last 2 terms in Eq. (4), respectively, gives the 2 terms in Eq. (5).

³ In Ref. [7], the definition of the index has to be amended by a factor of 1/2 in order to be consistent with Eq. (4) and most other definitions in the mathematics literature.

Table 2. The *exotica* with their respective gauge groups.

(a) $SU(3)_c \times SU(2)_L \times U(1)_Y$							
\tilde{Q}	$(\mathbf{3}, \mathbf{1})_{1/3}$	\tilde{E}_-	$(\mathbf{1}, \mathbf{1})_{-1}$	\tilde{L}	$(\mathbf{1}, \mathbf{2})_0$	\tilde{E}_\pm	$(\mathbf{1}, \mathbf{1})_{\pm 1}$
\tilde{Q}^c	$(\bar{\mathbf{3}}, \mathbf{1})_{-1/3}$	\tilde{E}_+^c	$(\mathbf{1}, \mathbf{1})_1$	\tilde{L}^c	$(\mathbf{1}, \mathbf{2})_0$	\tilde{E}_\pm^c	$(\mathbf{1}, \mathbf{1})_{\mp 1}$

(b) $SU(3)_c \times SU(2)_L \times SU(2)_A \times U(1)_Y$			
$2 \times (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1})_{-1/3}$	$2 \times (\mathbf{1}, \mathbf{1}, \mathbf{1})_1$	$2 \times (\mathbf{1}, \mathbf{2}, \mathbf{1})_0$	$2 \times (\mathbf{1}, \mathbf{1}, \mathbf{1})_{\pm 1}$
$2 \times (\mathbf{3}, \mathbf{1}, \mathbf{1})_{1/3}$	$2 \times (\mathbf{1}, \mathbf{1}, \mathbf{1})_{-1}$	$2 \times (\mathbf{1}, \mathbf{2}, \mathbf{1})_0$	$2 \times (\mathbf{1}, \mathbf{1}, \mathbf{2})_{\pm 1}$

(c) $SU(3)_c \times SU(2)_L \times SU(2)_A \times SU(2)_B \times U(1)_Y$			
$(\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{1})_{1/3}$	$5 \times (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{2/3}$	$2 \times (\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})_0$	$4 \times (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1})_{\pm 1}$
$(\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1}, \mathbf{1})_{-1/3}$	$5 \times (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{-2/3}$	$2 \times (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2})_0$	$4 \times (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{\pm 1}$

\tilde{Q}^c contributes the same amount, so in the end we have $\Delta b_3 = -1$, see Eq. (5).

The interesting point now is that for this special set of exotics, all Δb_i are equal:

$$\Delta b_3 = -1, \quad \Delta b_2 = -1, \quad \Delta b_1 = -1 \quad (9)$$

At the same time, the states in Tab. 2(a) are *not* in a **5** or **10** of $SU(5)$. In the following section 2.3, we will show that most predictions from Grand Unification are *not changed* in the presence of these states.

2.3 The Predictions from Grand Unification

Eq. (3) really corresponds to 3 equations (one for each value of the index $i = 1, 2, 3$) with 1 unknown M_{GUT} . We can e.g. take the first 2 equations, solve for M_{GUT} , use the relations $\alpha_{\text{em}} = \alpha_2 \sin^2 \theta = 3/5 \alpha_1 \cos^2 \theta$ and substitute $\mu = M_Z$:

1 The GUT Scale

$$M_{\text{GUT}} = M_Z \exp \left[\frac{2\pi}{b_2 - b_1} \frac{1}{\alpha_{\text{em}}(M_Z)} \left(\frac{3}{5} - \frac{8}{5} \sin^2 \theta(M_Z) \right) \right] \quad (10)$$

It turns out that M_{GUT} only depends on the *difference* between the b_i . Since our exotica contribute equally to the b_i (see Eq. (9)), the value of M_{GUT} is *unchanged* in the presence of these states.

2 The Strong Coupling Constant

$$\frac{1}{\alpha_3(M_Z)} = \frac{\sin^2 \theta(M_Z)}{\alpha_{\text{em}}(M_Z)} - \frac{b_3 - b_2}{2\pi} \log \left(\frac{M_{\text{GUT}}}{M_Z} \right) \quad (11)$$

Since α_3 is less well measured than the other coupling constants α_2 and α_1 , one usually solves for α_3 at the lower scale in terms of M_{GUT} and compares the low-energy prediction to experiment. Again, the result depends only on the difference of the b_i and thus cancels for the states in Tab. 2(a).

3 The Weinberg Angle

At the GUT scale, $\sin^2 \theta$ is determined by the relative normalization of $U(1)_Y$ to the Cartan generator in the GUT group which corresponds to hypercharge, see e.g. section 4.4 of Ref. [9] for a detailed discussion using the notation of the present publication. The low energy prediction for $\sin^2 \theta$ is not independent of the one for α_3 .

4 The GUT Coupling Constant

$$\frac{1}{\alpha_{\text{GUT}}} = \frac{\sin^2 \theta(M_Z)}{\alpha_{\text{em}}(M_Z)} + \frac{b_2}{b_2 - b_1} \frac{1}{\alpha_{\text{em}}(M_Z)} \left(\frac{3}{5} - \frac{8}{5} \sin^2 \theta(M_Z) \right) \quad (12)$$

The GUT coupling constant is the only prediction from Grand Unification which does change (namely it increases), since it does not only depend on the difference $b_2 - b_1$, but also directly on b_2 .

2.4 Connection to String Theory

We found the 3 sets of exotica given in Tab. 2 in the 5th twisted sector of the \mathbb{Z}_6 orbifold [10, 11, 12]. As a matter of fact, the states in Tab. 2(a) transform as

$$(\mathbf{6}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}) + \text{c.c} \quad (13)$$

under $SU(6) \times SU(2)$, before this symmetry is broken by Wilson lines to Pati-Salam and then spontaneously to the Standard Model.

In the minilandscape search [13, 14], we derived 127 MSSM-like models from string theory. It turns out that $\sim 5\%$ of these models contain exotics of the type listed in Tab. 2.

3 Phenomenological Consequences

3.1 Exotic Mesons and Baryons

Consider the exotica in Tab. 2(a) and notice that their electric charge $Q = T_3 + Y/2$ is fractional. As a consequence, the bound states of exotic quarks and SM quarks (see Tab. 3) will also have fractional electric charge.

We will assume that the exotic quarks have a gauge invariant, supersymmetric mass M of order the electroweak scale. Searches for fractionally charged heavy particles exclude them with mass less than 200 GeV [15, 16, 17]. Nevertheless they can be produced at the Tevatron or the LHC via Drell-Yan processes. For more details, cf. Ref. [3].

3.2 Exotica with Hidden Sector Charge

In string theory the exotica may transform non-trivially under a hidden sector gauge group. Here we consider a generalized example, not obtained from a particular

Table 3. The exotica form (fractionally charged) mesons and baryons. The superscripts denote electric charge.

(a) Exotic mesons.

$Q_u^{+1/2}$	$Q_d^{-1/2}$
$[\tilde{Q}u]_{+1/2}$	$[\tilde{Q}d]_{-1/2}$

(b) Exotic baryons.

$\Sigma_Q^{+3/2}$	$\Sigma_Q^{+1/2}$	$\Sigma_Q^{-1/2}$	$\Lambda_Q^{+1/2}$
$[\tilde{Q}uu]_{3/2}$	$[\tilde{Q}(ud)_s]_{1/2}$	$[\tilde{Q}dd]_{-1/2}$	$[\tilde{Q}(ud)_a]_{1/2}$

string construction, which has interesting phenomenology. Consider a hidden sector gauge group $SU(N)$ with the exotica transforming as

$$[(\mathbf{6}, \mathbf{1}, \mathbf{N}) + (\mathbf{1}, \mathbf{2}, \mathbf{N})] + \text{c.c.} \quad (14)$$

under $SU(6) \times SU(2)_R \times SU(N)$. Note that values of $N > 3$ generically give too large a value for α_{GUT} (see Eq. (12) and the following discussion) and are thus excluded by demanding perturbative unification.

Assuming the hidden sector gauge coupling gets strong at a scale $\Lambda_N \gg M_Z$, the exotica will form $SU(N)$ singlet bound states with mass of order Λ_N , and the phenomenology of such $SU(N)$ singlet “baryons” and “mesons” will depend on the values of N and Λ_N .

Current bounds restrict us to a gauge invariant mass $M \gtrsim 200$. We then can consider two possibilities, either $\Lambda_N \geq M$ or $\Lambda_N \ll M$. The first case is comparable to QCD with all quark masses less than or equal to Λ_{QCD} . The second case is more interesting. The exotica will have properties similar to the “quirks” introduced in Ref. [18]. They can be produced at the LHC. When the exotics are produced they can separate by large distances in the detector before forming the bound state, since their effective string tension is so much smaller than their mass. Again, cf. Ref. [3] for more details.

4 Conclusions

Supersymmetry and Grand Unification are prime candidates for physics beyond the SM. As such, we would very much like to keep their predictions as guidelines for model building. It is well known that states that come in complete multiplets of the grand unified gauge group do not affect gauge coupling unification. In this study, we have discussed a novel feature of light exotic particles

- (i) that do not come in complete $SU(5)$ multiplets,
- (ii) that do not affect gauge coupling unification and most other predictions of GUTs (at one loop),
- (iii) whose only effect is to increase the value of α_{GUT} ,
- (iv) that have fractional electric charge,
- (v) that are found in orbifold constructions of the heterotic string.

Clearly, all these exotica would have very distinctive signatures at the LHC.

Acknowledgments. We are indebted to the Ohio Supercomputing Center for using their resources and to Ben Dundee and Hasan Yüksel for useful discussions. We would like to acknowledge research supported in part by the Department of Energy under Grant No. DOE/ER/01545-872.

References

1. S. Dimopoulos, S. Raby and F. Wilczek, *Phys. Rev. D* **24**, 1681 (1981); S. Dimopoulos and H. Georgi, *Nucl. Phys. B* **193**, 150 (1981); L. Ibanez and G.G. Ross, *Phys. Lett. B* **105**, 439 (1981); N. Sakai, *Z. Phys. C* **11**, 153 (1981); M. B. Einhorn and D. R. T. Jones, *Nucl. Phys. B* **196**, 475 (1982); W. J. Marciano and G. Senjanovic, *Phys. Rev. D* **25**, 3092 (1982).
2. See review article, Grand Unified Theories, S. Raby, Particle Data Group, W.-M. Yao et al., *Journal of Physics G* **33**, 1 (2006).
3. S. Raby and A. Wingerter, *Phys. Rev. Lett.* **99**, 051802 (2007) [arXiv:0705.0294 [hep-ph]].
4. W. M. Yao et al. [Particle Data Group], *J. Phys. G* **33**, 1 (2006).
5. H. D. Politzer, *Phys. Rev. Lett.* **30**, 1346 (1973).
6. D. J. Gross and F. Wilczek, *Phys. Rev. Lett.* **30**, 1343 (1973).
7. R. Slansky, *Phys. Rept.* **79**, 1 (1981).
8. P. Di Francesco, P. Mathieu and D. Senechal, *New York, USA: Springer (1997) 890 p*
9. S. Raby and A. Wingerter, arXiv:0706.0217 [hep-th].
10. T. Kobayashi, S. Raby and R. J. Zhang, *Phys. Lett. B* **593**, 262 (2004) [arXiv:hep-ph/0403065].
11. T. Kobayashi, S. Raby and R. J. Zhang, *Nucl. Phys. B* **704**, 3 (2005) [arXiv:hep-ph/0409098].
12. W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, *Phys. Rev. Lett.* **96**, 121602 (2006) [arXiv:hep-ph/0511035].
13. O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange and A. Wingerter, *Phys. Lett. B* **645**, 88 (2007) [arXiv:hep-th/0611095].
14. O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange and A. Wingerter, arXiv:0708.2691 [hep-th].
15. D. Acosta et al. [CDF Collaboration], *Phys. Rev. Lett.* **90**, 131801 (2003) [arXiv:hep-ex/0211064].
16. M. L. Perl, E. R. Lee and D. Loomba, *Mod. Phys. Lett. A* **19**, 2595 (2004).
17. M. Fairbairn, A. C. Kraan, D. A. Milstead, T. Sjostrand, P. Skands and T. Sloan, *Phys. Rept.* **438**, 1 (2007) [arXiv:hep-ph/0611040].
18. J. Kang, M. A. Luty and S. Nasri, arXiv:hep-ph/0611322.